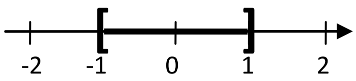
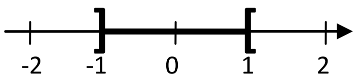
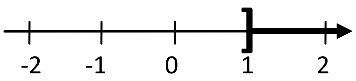
FUNKTIONEN

Intervalle

$[-1;1]$	$-1 \le x \le 1$	$]-1;1[$	$-1 < x < 1$	$]1;\infty[$	$1 < x < \infty$
					

Polynom-Division

$(x^2+2x+1)/(x+1) = x+1$

$\underline{x^2 + x}$

$x+1$

$\underline{x+1}$

0

$y(x) = (x+1)(x+1)$

$(x^4 - x^3 + x^2 - x + a)/(x-2) = x^3 + x^2 + 3x + 5$

$\underline{x^4 - 2x^3}$

$x^3 + x^2 - x + a$

$\underline{x^3 - 2x^2}$

$3x^2 - x + a$

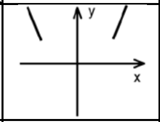
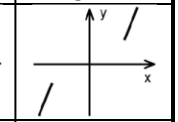
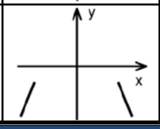
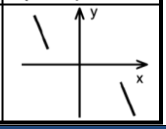
$\underline{3x^2 - 6x}$

$5x + a$

$\underline{5x - 10}$

$a + 10 = 0 \quad \rightarrow \quad \underline{a = -10}$

Allgemeine Polynom-Form

a_n \ n	Gerade	Ungerade
> 0		
< 0		

allgemeine Polynom

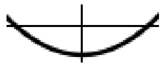
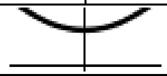
$$y = a_n x^n + \cdots + a_1 x + a_0$$

$$y = \sum_{i=0}^{n} a_i x^i$$

Linearfaktor-Ansatz

$$y = a(x - x_1)^z (x - x_2)$$

Quadratische Funktion

Normalform	$y = ax^2 + bx + c$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ → Diskriminante			
Scheitelform	$y = a(x-c)^2 + d$	Scheitelpunkt $S(c\|d)$			
Linearfaktor	$y = a(x-x_1)(x-x_2)$	Linearfaktoren $x_i = x_1, x_2$			
Diskriminante	$D = b^2 - 4ac$	> 0		$x_1 \neq x_2$	$x_i = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
		$= 0$		$x_1 = x_2$	$x_i = -\frac{b}{2a}$
		< 0		$x_1, x_2 \in \mathbb{C}$	$x_i = -\frac{b}{2a} \pm j\frac{\sqrt{4ac - b^2}}{2a}$

Exponentialfunktion

$y = a * q^{b*x+c} + d$
$y = A * B^x$
$y = A * e^{\lambda * x}$

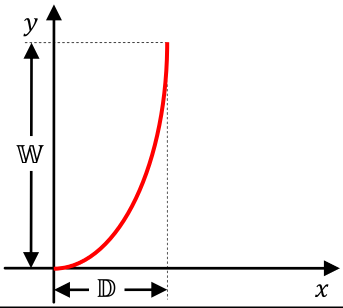
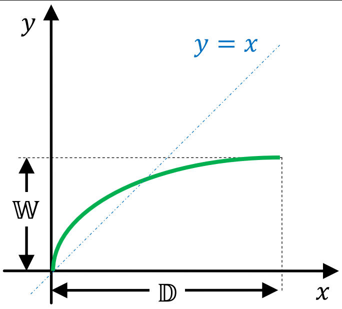
a, A	Anfangswert	
b, λ	Zeit bis Vervielfachung ($-\lambda = Abnahme$)	
q, B, e	Wachstumsfaktor $\left(1 + \frac{p\,(\text{Zins})}{100}\right)$	$> 1 = Zunahme$ $< 1 = Abnahme$
c	Zeit auf null setzen	
d	Nicht vervielfachender Wert	

Trigonometrische Funktionen

$y = a * sin(bx + c) + d$
$y = a * cos(bx + c) + d$
$y = a * tan(bx + c) + d$

a = Amplitude
b = Kreisfrequenz
c = Phasenverschiebung
Periodendauer $= \frac{2\pi}{b}$

Umkehrfunktion (=Inverse) $f^{-1}(x)$

Funktion $f: y = f(x)$	$x \to y$	Funktion oben minus eins $f^{-1}: x = f^{-1}(y)$	$y \to x$
	$f: y(x) = x^2$ $\mathbb{D} = \{x \mid 0 \le x \le 2\}$ $\mathbb{W} = \{y \mid 0 \le y \le 4\}$		$f^{-1}: y(x) = \sqrt{x}$ $\mathbb{D} = \{y \mid 0 \le y \le 4\}$ $\mathbb{W} = \{x \mid 0 \le x \le 2\}$
invertierbar, umkehrbar: wenn $f'(x)$ existiert.		geometrisch gesehen eine Spiegelung an $y = x$	

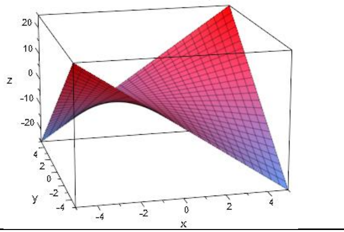
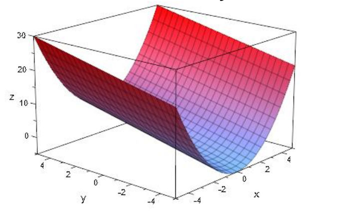
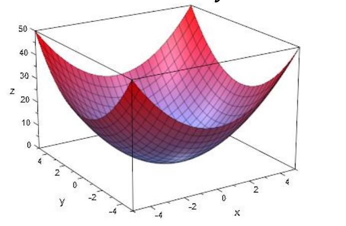
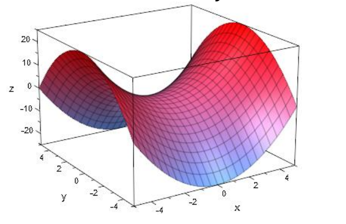
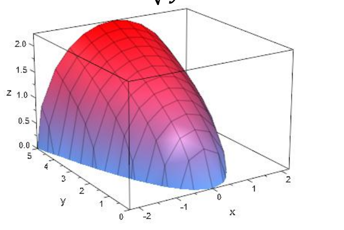
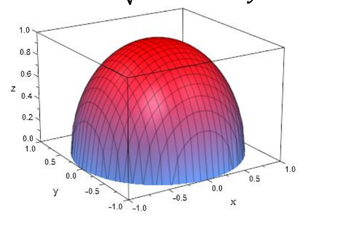
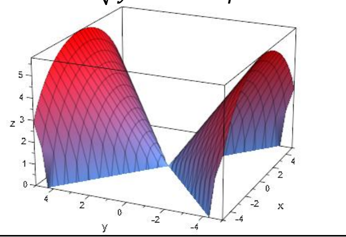
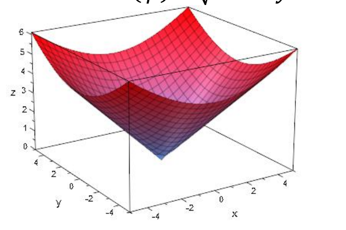
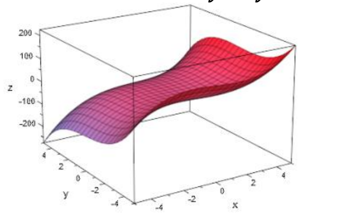
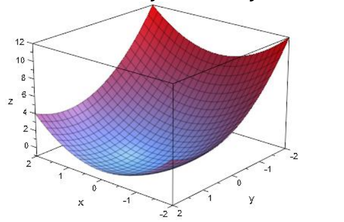
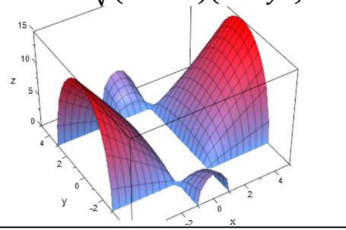
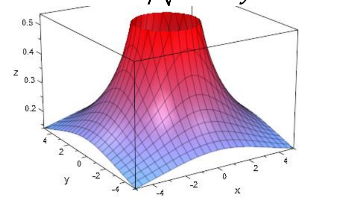
Umkehrfunktionen der Trigonometrischen Funktionen

$f(x) = \sin \alpha$		$f^{-1}(x) = \arcsin \alpha$
$f(x) = \cos \alpha$		$f^{-1}(x) = \arccos \alpha$
$f(x) = \tan a$	$\frac{1}{f(x)} = \cot a$	$f^{-1}(x) = \arctan a$

Trigonometrische Funktionen in Komplexe Zahlen

$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$	$\sinh x = \frac{e^{x} - e^{-x}}{2}$
$\cos x = \frac{e^{ix} + e^{-ix}}{2i}$	$\cosh x = \frac{e^{x} + e^{-x}}{2}$

Beispiele von Funktionen

$z = x * y$	$z = x^2 + y$	$z = x^2 + y^2$	$z = x^2 - y^2$
			
$z = \sqrt{y - x^2}$	$z = \sqrt{1 - x^2 - y^2}$	$z = \sqrt{y^2 * \tan^2 \varphi - x^2}$	$z = \cot(\varphi) * \sqrt{x^2 + y^2}$
			
$z = x^3 + x * y - y^3$	$z = x^2 + y^2 - 2 * y * x$	$z = \sqrt{(x^2 - 1)(9 - y^2)}$	$z = 1/\sqrt{x^2 + y^2}$
			

Wichtige Funktionen

Kugel	**Einheitskugel**	**Kreis (kartesisch)**	**Kreis (polar)**	**Einheitskreis**	**Ellipse**
$R^2 = x^2 + y^2 + z^2$	$1 = x^2 + y^2 + z^2$	$R^2 = x^2 + y^2$	$x(t) = R * \cos(t)$ $y(t) = R * \sin(t)$ $\quad 0 \le t \le 2\pi$	$1 = x^2 + y^2$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$