LAPLACE-TRANSFORMATION

($\mathcal{L}-Operator$)

Laplace-Transformation

$$G(s) = \int_0^{\infty} g(t) * e^{-st}\,dt$$

Funktion im Zeitbereich
$g(t) = \mathcal{L}^{-1}[G(s)]$

○—●

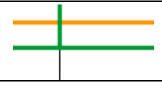
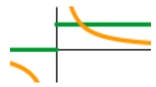
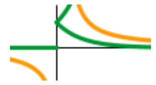
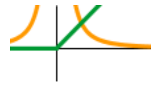
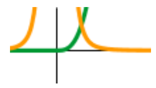
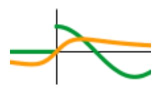
Funktion im Bildbereich
$G(s) = \mathcal{L}[g(t)]$

$g(t) = 0\ für\ t < 0$
$t \in \mathbb{R}$

Inverse Laplace-Transformation

$s = \sigma + i\omega$
$s \in \mathbb{C}$

Reduzierte Transformationstabelle

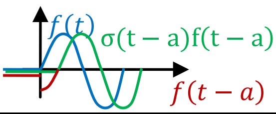
$g(t)$	○—●	$G(s)$
Dirac $\delta(t)$		1
Heavyside'sche Sprungfunktion $1 = \sigma(t)$		$\frac{1}{s}$
e^{at}		$\frac{1}{s-a}$
t		$\frac{1}{s^2}$
$\frac{t^{n-1}}{(n-1)!}$		$\frac{1}{s^n}$
$\frac{1}{a}\sin(at)$		$\frac{1}{s^2+a^2}$
$\cos(at)$		$\frac{s}{s^2+a^2}$

Rezept bezüglich Nullstellen

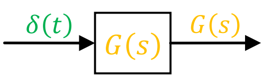
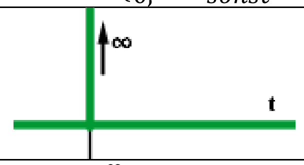
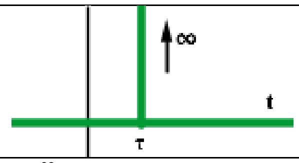
$$Typ: \frac{P_1(s) \rightarrow beliebig}{P_2(s) \rightarrow Polynom\ 2.Grades}$$

Nullstellen	Vorgehen	Ergebnis
$x_0 = 2 \in \mathbb{R}$	1. Partialbruchzerlegung	e^{at}
$x_0 = 1 \in \mathbb{R}$ $(doppelt)$	a. Faltungssatz wenn Zähler min linear b. Dämpfungssatz wenn Zähler konstant	e^{at}
$x_0 = 2 \in \mathbb{C}$	1. $\frac{s+2}{(s^2+2s+5)} \rightarrow \frac{s+1}{(s+1)^2+2^2} + \frac{1}{(s+1)^2+2^2}$ 2. Dämpfungssatz für $s \Leftrightarrow (s+1)^2$ $\frac{s}{s^2+2^2} + \frac{1}{s^2+2^2}$ ○—● $\cos(2x) + \frac{1}{2}\sin(2t)$	sin, cos

Regeln

Linearitätssatz	$a * f(t) + b * g(t)$ Addition	○—● ○—●	$a * F(s) + b * G(s)$ Addition
Verschiebungssatz 	$\sigma(t-a) * f(t-a)$ Verschiebung n. rechts	○—● ○—● $a > 0$	$e^{-as}F(s)$ Dämpfung
Dämpfungssatz	$e^{-at} * f(t)$ Dämpfung	○—● ○—●	$F(s+a)$ Verschiebung n. links
Faltungssatz	$f(t) \star g(t)$ Faltung $Faltung\!: h(t) = f(t) \star g(t) = \int_0^t f(\tau)g(t-\tau)d\tau$	○—● ○—●	$F(s) * G(s)$ Multiplikation
Differentiationssatz	$f'(t)$ ○—● $f''(t)$ ○—● $f'''(t)$ ○—●		$s * F(s) - f(+0)$ $s^2 * F(s) - s * f(+0) - f'(+0)$ $s^3F(s) - s^2f(+0) - sf'(+0) - f''(+0)$
Integrationssatz	$\int_0^t f(\tau)d\tau$	○—●	$\frac{F(s)}{s}$
Grenzwertsätze „Anfangswerte haben einen Bezug zum Limes"	$\exists \lim_{t\to\infty} f(t)$ $\lim_{\substack{t\to 0\\ t>0}} f(t) = \lim_{s\to\infty} s * F(s)$,	$\exists \lim_{\substack{t\to 0\\ t>0}} f(t)$ $\lim_{t\to\infty} f(t) = \lim_{s\to 0} s * F(s)$	$\exists\, f'(t)\ \forall\, t > 0$

Dirac-Stoss

Eigenschaften Keine Funktion, sondern eine Distribution Praktisch: kurzer, fester Schlag **Verwendung**: zur Systemidentifikation 	$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & sonst \end{cases}$  $\int_{-\infty}^{\infty} \delta(t) = 1$	$\delta(t-\tau)$  $\int_{-\infty}^{\infty} \delta(t-\tau)f(t) = f(\tau)$