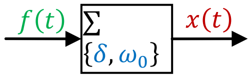
LINEARE SCHWINGUNGEN

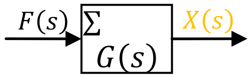
Schwingungsgleichung

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = f(t)$$

δ: Dämpfungskonstante ($\delta > 0$)
ω_0: Eigenfrequenz ($\omega_0 > 0$)



Freie Schwingung $f(t) = 0$



Erzwungene Schwingung $f(t) = k_0 * \sin \omega t$
schwach gedämpftes System: $0 < \delta < \omega_0$

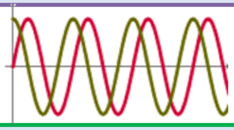
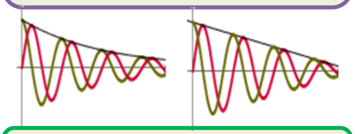
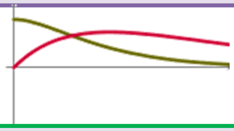
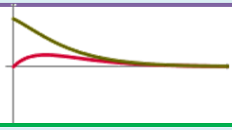
allgemeine Lösung

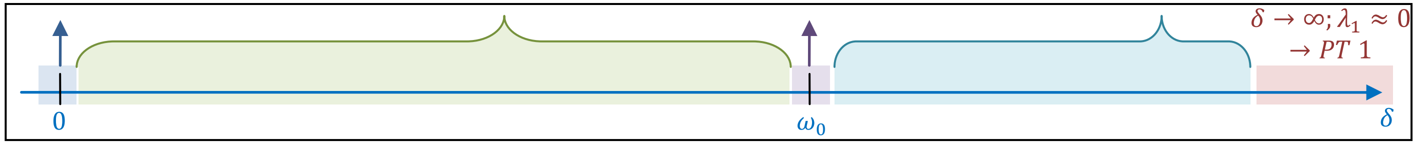
$$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}, \quad \lambda_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} \quad ○\!\!-\!\!\!-\!\!\bullet \quad X(s) = \frac{1}{s^2 + 2\delta s + \omega_0^2}$$

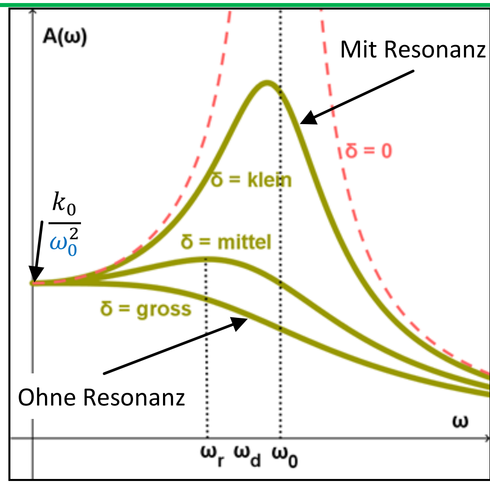
Lösung

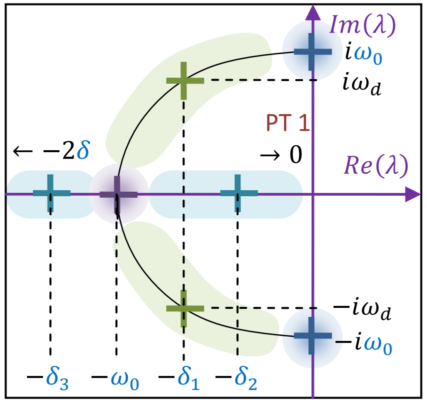
$$A(\omega) = \frac{k_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2}}$$

$$\varphi(\omega) = \begin{cases} \tan^{-1}\frac{2\delta\omega}{(\omega_0^2 - \omega^2)^2} & , \omega < \omega_0 \\ \pi/2 & , \omega = \omega_0 \\ \tan^{-1}\frac{2\delta\omega}{(\omega_0^2 - \omega^2)^2} + \pi & , \omega > \omega_0 \end{cases}$$

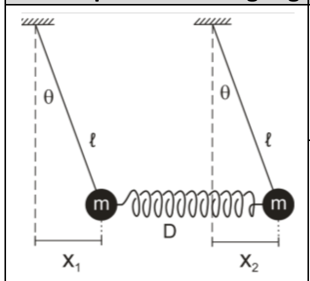
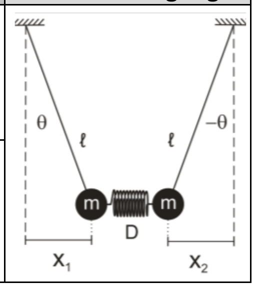
ungedämpfte Schwingung $\delta = 0$	schwach gedämpfte Schwingung $0 < \delta < \omega_0$	aperiodischer Grenzfall $\delta = \omega_0$	starke Dämpfung (Kriechfall) $\delta > \omega_0$
			
Lösung $C_1 \sin \omega_0 t + C_2 \cos \omega_0 t$	**Lösung** $e^{-\delta t}(C_1 \sin \omega_d t + C_2 \cos \omega_d t)$	**Lösung** $(C_1 t + C_2) * e^{-\delta t}$	**Lösung** $C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$
$\lambda_{1,2} = \pm i\omega_0$	$\lambda_{1,2} = -\delta \pm i\sqrt{\omega_0^2 - \delta^2}$	$\lambda_{1,2} = -\delta$	$\lambda_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$







Gekoppelte Schwingung

Schwerpunktsschwingung		Relativschwingung
	$x_s = \frac{x_1 + x_2}{2}$ $x_r = \frac{x_1 - x_2}{2}$ $x_1 = x_s + x_r$ $x_2 = x_s - x_r$	

Frequenzen
ungedämpftes System $\omega = \omega_0 = 2\pi f$
gedämpftes System $\omega_d = \sqrt{\omega_0^2 - \delta^2}$
Resonanzfrequenz $\omega_r = \sqrt{\omega_0^2 - 2\delta^2}$