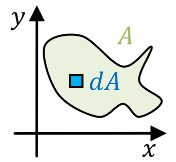
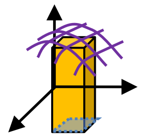
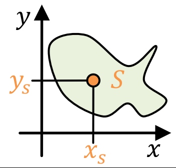
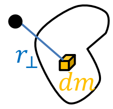
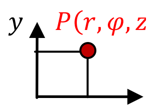
ZWEIFACH- UND DREIFACHINTEGRALE

Zweifachintegrale

Flächenberechnung		Volumen	Schwerpunkt S		Trägheitsmoment
Ebene	Oberfläche O_b		homogen	inhomogen	J_z
$A = \int_A 1\, dA$	$= \int_A \sqrt{{f_x}^2 + {f_y}^2 + 1}\, dA$	$V = \int_A f(x,y)\, dA$	$x_s = \frac{1}{A}\int_A x\, dA$ $y_s = \frac{1}{A}\int_A y\, dA$	$x_s = \frac{1}{M}\int_A x\,\rho(x,y)\, dA$ $y_s = \frac{1}{M}\int_A y\,\rho(x,y)\, dA$	$= \delta \int_A (x^2+y^2)\, dA$
				$Dichte: [\rho] = \frac{kg}{m^2}$ $M = \int_A \rho(x,y)\, dA$	

Achten, ob Integral im Positiven bleibt!! Integrale von Innen nach Aussen lösen

Kartesische Koordinaten



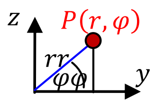
$dx\Delta A = \Delta x * \Delta y$

dy
dx

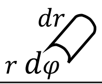
$r = \sqrt{x^2+y^2}$
$\tan\varphi = {}^{y}/_{x}$

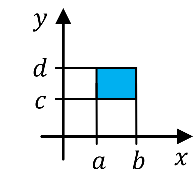
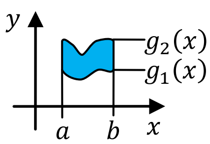
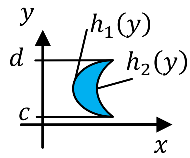
$x = r * \cos\varphi$
$y = r * \sin\varphi$

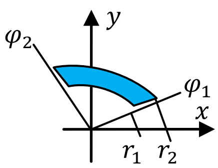
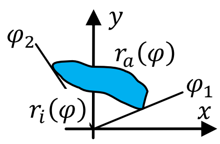
Polar Koordinaten



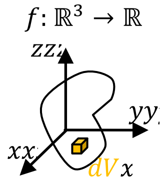
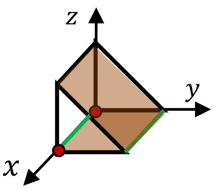
$\Delta A = r * dr * d\varphi$



Allgemein	$y = g(x)$	$x = h(y)$
$V = \int_a^b \int_c^d f(x,y)\, dy\, dx$	$V = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx$	$V = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dx\, dy$
		
vertauschbar	nicht vertauschbar	nicht vertauschbar

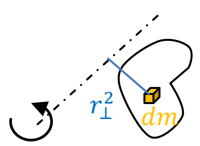
$V = \int_{\varphi_1}^{\varphi_2} \int_{r_1}^{r_2} f(r,\varphi)\, r\, dr\, d\varphi$	$V = \int_{\varphi_1}^{\varphi_2} \int_{r_i(\varphi)}^{r_2(\varphi)} f(r,\varphi)\, r\, dr\, d\varphi$
	
vertauschbar	nicht vertauschbar

Dreifachintegrale

Allgemein	Ausrechnen von Dreifachintegralen		
$V = \int_V f(x,y,z)\, dV$	$V = \int_0^1 \int_0^1 \int_0^{1-y} 1\, dz\, dy\, dx$		
$f: \mathbb{R}^3 \to \mathbb{R}$ 		Punkte: $x = 0$	$x = 1$
		Linien: $y = 0$	$y = 1$
		Flächen: $z = 0$	$z = 1 - y$

Volumen	$V = \int_V 1\, dV$
homogene Masse	$M = \int_V \rho(x,y,z)\, dV$
potenzielle Energie	$E = \int_V dE = \delta\, g \int_V z\, dV$

Grenzen dürfen nur Variablen äusserer Integrale besitzen.

Massenträgheitsmoment			Schwerpunkt	
allgemein	homogener	inhomogen	homogen	inhomogen
$J = m * r_\perp^2 = \int_V dJ\, dV$	$J = \delta \int_V r_\perp^2\, dV$	$J = \int_V r_\perp^2\, \delta(x,y,z)\, dV$	$x_s = \frac{1}{V}\int_V x\, dV$	$x_s = \frac{1}{M}\int_V \rho(x,y,z)\, x\, dV$
	$J_z = \delta \int_V (x^2+y^2)\, dV$	$J_z = \int_V (x^2+y^2)\, \delta(x,y,z)\, dV$	$y_s = \frac{1}{V}\int_V y\, dV$	$y_s = \frac{1}{M}\int_V \rho(x,y,z)\, y\, dV$
	$J_x = \delta \int_V (y^2+z^2)\, dV$	$J_x = \int_V (y^2+z^2)\, \delta(x,y,z)\, dV$	$z_s = \frac{1}{V}\int_V z\, dV$	$z_s = \frac{1}{M}\int_V \rho(x,y,z)\, z\, dV$
	$J_y = \delta \int_V (x^2+z^2)\, dV$	$J_y = \int_V (x^2+z^2)\, \delta(x,y,z)\, dV$		